Explanation in variable phonology: An exponential model of morphological constraints

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ABSTRACT

Variationist treatments of phonological processes typically provide precise quantitative accounts of the effects of conditioning environmental factors on the occurrence of the process, and these effects have been shown to be robust for several well-studied processes. But comparable precision in theoretical explanation is usually elusive, at the current state of the discipline. That is, the analyst is usually unable to say why the parameters should have the particular values that they do, although one can often explain relative ordering of environments. This article attempts to give a precise explanation—in the form of a quantitative theoretical prediction—of one robust quantitative observation about English phonology. The reduction of final consonant clusters (often called -t,d deletion) is well-known to be conditioned by the morphological structure of a target word. Deletion applies more in monomorphemic words (e.g., mist) than in inflected words (e.g., missed). In the theory of lexical phonology, these classes of words are differentiated by derivational history, acquiring their final clusters at different levels of the morphemes. The theory further postulates that rules may apply at more than one level of the derivation. If -t,d deletion is treated as a variable rule with a fixed rate of application (p_0) in a phonology with this architecture, then higher rates of application in underived forms (where the final cluster is present underlyingly and throughout the derivation) are a consequence of multiple exposures to the deletion rule, whereas inflected forms (which only meet the structural description of the rule late in the derivation) have fewer exposures and lower cumulative deletion. This further allows a precise quantitative prediction concerning surface deletion rates in the different morphological categories. They should be related as an exponential function of p_0, depending on the number of exposures to the rule. The prediction is empirically verified in a study of -t,d deletion in seven English speakers.

Studies of linguistic variation have achieved a high level of quantitative precision in describing the systematic patterns of "orderly heterogeneity" (Weinreich, Labov, & Herzog, 1968:100) that permeate human use of language. Using the computational and descriptive apparatus of the variable rule frame-

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work (Cedergren & Sankoff, 1974; Labov, 1969; Rousseau & Sankoff, 1978; Sankoff & Labov, 1979), one can measure rates of occurrence of variable processes, describe the effects on such processes of conditioning factors, and model the interaction of multiple independent variables in giving rise to observed rates of use of the variants. Numerical measures of factor strengths and of statistical significance can be given to three or more significant digits. With these techniques and an impressive body of empirical studies completed, the field can be said to have achieved a certain level of descriptive adequacy.

This descriptive precision is not generally matched, however, by explanatory precision. That is, the analyst usually cannot say why the quantitative values obtained should have the particular values that they do. It can often be predicted, on theoretical grounds, that factor weights should have a particular rank order of values, but this is generally the acme of quantitative explanation. Theories that predict particular quantitative values for linguistic variables are in very short supply in linguistics. The development of models that have explanatory value in this sense—models from which one can derive precise quantitative predictions—is one of the fundamental challenges facing our discipline.

The present work attempts to make a contribution to this undertaking, by sketching a model of a variable phonological process of English that makes possible precise quantitative predictions about frequency of occurrence of the process. As I show, these predictions are empirically confirmed in a study of production. The process in question is the simplification of final consonant clusters that has been shown to occur in most English dialects; typically this involves loss of final /t/ or /d/. Numerous studies have shown that the process is morphologically constrained (e.g., Fasold, 1972; Guy, 1980; Guy & Boyd, 1990; Labov, 1989; Labov, Cohen, Robins, & Lewis, 1968; Nesbitt, 1984; Neu, 1980; Wolfram, 1969). In the speech of native speakers, underived or monomorphemic words such as mist, pact, undergo deletion at a higher rate than inflected forms such as past tense verbs like missed, packed. Within the inflected words, a further differentiation is found for many adult speakers between two morphological classes of verbs. The regular or weak past tense forms undergo deletion less frequently than the semiweak verbs, which have a vowel change in the past tense in addition to an apical stop suffix, for example, left, kept, told. Thus, morphological properties of words in some way influence the operation of a phonological process.

The problem, then, is to characterize the nature of this influence. Traditionally, variationist studies of this phenomenon (including my own) have treated the morphological category as an independent constraint on the operation of a variable deletion rule—a factor group consisting of three factors. The ordering of the factor values in this group have been “explained” in terms of two approaches. One involves a functional explanation. The in-
flectional -t,ds, representing the past tense morpheme, have a higher functional load than the noninflectional ones and are accordingly more resistant to deletion, and the difference between semiweak verbs and regular verbs is due to the functional difference in the affix. In walked, talked, and so forth, the affix is the unique marker of past tense, while in left, told, it is not unique. A second explanation appeals directly to the difference in morphological structure as a constraint on the rule. Thus, the presence (and nature) of a morphological boundary can be characterized as a constraining factor on the operation of the variable deletion rule, without reference to the functional significance of the segments involved. One can stipulate that the rule is disfavored by a boundary before the final stop and that the /#/ boundary is more disfavoring than the /+/- boundary.

Both of these “explanations” are subject to the same limitation. They provide no explicit quantitative prediction as to the strengths of the factors involved—the actual quantifiable rates of deletion observed in empirical studies. At best, they yield an ordering of the constraint values: monomorphemic > semiweak past tense > regular past tense (where “>” is to be read as “favors deletion more than”). This ordering is consistent with the empirical findings, but it does not answer questions like: Should the deletion rate in semiweak verbs be closer to monomorphemic words or regular past tense forms? Should the rate of deletion in regular past tense words be some function of the rate in monomorphemic words, such as (1/M) or (M − C)? The explanations given earlier are silent on this point. To my knowledge, no one has proposed a quantified theory that permits one to make statements of the form: the /#/ boundary in regular verbs (or the higher functional load of the affix) has an inhibitory value X that predicts a value Y for -t,ds deletion in this category and that differs quantifiably from inhibitory value Z of the /+/- boundary in semiweak verbs. Furthermore, the currently available theories shed no light on the question of whether morphological constraints are any different in their quantitative behavior from other kinds of constraints, such as the following segment effect on -t,ds deletion.

I account for morphological differences in -t,ds deletion not in terms of a discrete morphological constraint on the process but as a consequence of a particular organization of the morphology and phonology. I adopt this organization from the theory of lexical phonology (Kiparsky, 1982, 1985; Mohanan, 1986). In this approach, these categories differ in terms of derivational history, specifically in terms of the derivational level at which they acquire the final cluster. Rules are assumed to be able to apply at more than one level of a derivation, so that some morphological categories may be repeatedly exposed to the operation of a rule. The details of the model are sketched later. They allow us to make a precise quantitative prediction about deletion rates in the three morphological categories: they should be related by an exponential function. As I show, an empirical study of the operation of the rule in the usage of seven English speakers confirms this prediction.
THE DATA

The data for the quantitative analyses presented were obtained from sociolinguistic interviews with seven native speakers of English. The consultants are diversified on most social dimensions and speak several different varieties of North American English; their social characteristics are given in the Appendix. The interviews and the extraction and coding of the data were undertaken by students in the Linguistics 257 course at Stanford University in the spring of 1990. I express my appreciation to these researchers: Renee Blake, Erica Deese-Dobson, Gregório Firmino, Rudi Gaudio, Martha Porras, Jeff Seinfeld, and Mark Van Haren.

Each consultant was interviewed for a period of 30–60 minutes, and the first 100–200 relevant tokens (in most cases, this meant all) were extracted from the tape recording of the interview. Relevant tokens were defined as including any word terminating in a consonant cluster with a final -t or -d, except for certain high-frequency items about whose underlying representation there is some uncertainty: contractions of not (e.g., wasn’t, won’t) and the conjunction and. Tokens in a neutralizing environment (i.e., immediately followed by a word beginning with another apical stop, such as best time) were not included in the quantitative analysis.

Each token was coded impressionistically as showing retention or deletion of the final stop; forms showing any overt reflex of the stop were treated as retained (including glottal stops, and affricates derived from stop + glide sequences). All data were also coded for the three independent contextual variables ("factor groups") described later and stored as token files. Data entry, file manipulation, and variable rule analysis were conducted using the MacVarb application developed by Guy and Lipa (see Guy, 1989), implementing a version of Varbrul 2S on the Macintosh microcomputer.

The factor groups used in the coding conformed to the practice of previous studies of the process, as follows. First, each token was classified according to the three morphological types described: as a monomorphemic form (which category included strong verbs and suppletive forms, such as found and went), a past tense form of a semiweak verb, or a past tense form of a regular verb. (In what follows, I occasionally refer to these three classes with the abbreviations M, S, and P, respectively.) In addition, two phonological factor groups were utilized in the study. The manner of the consonant preceding the -t or -d was coded as stop, sibilant, nonsibilant fricative, nasal, or liquid, and the following phonological context was coded as obstruent consonant (fricatives and oral or nasal stops), lateral, rhotic, glide, vowel, or nothing (in utterance-final and prepausal cases). (These phonological conditioning factors receive little attention in what follows, for reasons of brevity; they are cited here for the sake of completeness and replicability.)

The pattern of morphological conditioning that I wish to account for is illustrated in the pooled data in Table 1, showing rates of -t,d absence in the
three morphological categories. These results are comparable with those of the previous studies cited earlier.

THE THEORETICAL MODEL

The basic problem, then, is to account for the findings in Table 1. The solution proposed here involves a synthesis of elements of variable rule analysis and lexical phonology. In this section, I delineate the relevant features and the way in which I am making use of them.

Variable rule analysis

The theoretical framework of variable rule analysis has been characterized by several fundamental assumptions since it was proposed by Labov (1969) and formalized in works such as Cedergren and Sankoff (1974) and Sankoff (1978). Three of these are important for my purposes:

1. Individual speakers may differ in their basic rate of use of a variable rule, that is, in their “input probability” for the rule.
2. Individuals should be similar or identical in the factor values assigned to linguistic constraints on the rule. (This assumption is usually qualified to apply just to people who belong to the same speech community.)
3. The factor values, or conditional probabilities associated with each environmental constraint on a rule, can freely vary in the interval (0–1) (subject in the logistic model only to the definitional requirement that the factors in a group average to .5). In other words, the variable rule framework imposes no a priori requirement on how close or distant the values in a given group may be or on what ratios obtain between them. In fact, the mathematical model used for constraint estimation is specifically agnostic on these issues. It can equally well model a factor group whose values are closely approximated at .51 and .49 or a group whose values are wildly divergent, such as .99 and .01.
The first of these assumptions, that individuals may differ in their base rate of rule application, I adopt without further discussion. The second, that individuals may not differ in factor values, I adopt in modified form. First, I assume that some of the relationships among factor values are a consequence of essentially universal properties, such as the favoring of consonantal deletion before another consonant more than before a following vowel. These orders should therefore not be perturbed by any speaker. But other factor weights are evidently due to language- or dialect-specific properties, such as the dialectal differences in the effect of pause on deletion rules demonstrated in Guy (1980). In such cases, significant differences in the values of a factor will be taken as indicative of real differences in the mental grammar.

The principal focus of this article is the third assumption, that of the unconstrained nature of the ratios among different factors in a group. These are treated in variable rule analysis as an empirical issue. No prediction is made on theoretical grounds as to how much more deletion of final consonants there should be before a consonant than before a vowel; nor, for example, in English copula contraction, what the ratio of contraction after a pronoun should be to the rate after a full NP. The practice in the field is usually to find out what values are obtained from an empirical study and then "explain" the values post-hoc. Used in this way, the mechanics of variable rule analysis become a discovery device, or a kind of summary statistic. But to use this device to test a hypothesis, or to pursue an explanation of the phenomena observed, requires something else in addition to the variable rule mechanisms, namely, a model or theory of the elements and the events of language.

**Lexical phonology**

In the case at hand, a stronger model of events is obtained if we can enunciate principles that impose tighter constraints on the range of values allowed in a factor group, or the ratios that may exist among those values. Lexical phonology (see Kiparsky, 1982; Mohanan, 1986) provides an architecture that makes this possible. The features of this theory that are relevant to my argument can be summarized as follows:

1. multiple levels of lexical derivation;
2. interleaving of morphological and phonological processes;
3. phonological rules may apply at more than one level; and
4. bracket erasure occurs at the end of each level.

Briefly, the theory proposes that lexical derivation is a structured process organized into levels, with an interleaving of the applications of morphological and phonological rules. At each level, the morphological processes (like concatenation of affixes) alternate with the phonological processes, and a form must pass through all levels before surfacing. The principal level dis-
tinction contrasts lexical and postlexical processes. Within the lexicon, there is a further differentiation of at least two ordered levels at which different types of morphological processes operate. Some versions of the theory propose more than two lexical levels, but Harris (1989) found two levels are sufficient to account for patterns of variation and change in English. To simplify for present purposes, irregular inflection (that which is lexically specific and triggers exceptional morphophonemic rules) occurs at level 1 (along with certain types of affixation), whereas the attachment of regular inflectional affixes occurs at level 2 (along with certain other affixes and compounding processes).

A crucial characteristic of the theory is that the phonological rules are not necessarily assigned to specific levels. Some rules may be restricted to a single level, and there are various diagnostic tests for identifying such restrictions (e.g., rules that do not apply across word boundaries must operate only within the lexicon, not postlexically). However, it is possible for rules to apply at many levels. Each rule's domain of application must be stipulated; maximally general ones apply everywhere. Finally, the theory assumes that boundaries such as /+/ and /#/ do not exist; the only bracketing used is the demarcation of morphemes within a derivational level. At the end of each level, bracket erasure occurs, so that the internal structure of a form is not available to condition rules at later levels.

In the case of the morphological categories being treated here, monomorphemic words would have their final -t or -d underlingly, present from the earliest stage of the derivation. Semiweak verbs, because they undergo internal vowel changes and regressive voicing assimilation (leave-left) or no voicing assimilation (feel-felt), would be treated as undergoing affixation at level 1 of the lexicon; the weak verbs would have their regular affixal -t or -d inserted at level 2 of the lexicon. So, each has a distinct derivational history in this model, acquiring the final cluster that is the target of -t,d deletion at a different point.

What is the correct domain of application of -t,d deletion in such a framework? It is conditioned by the initial segment of a following word, which is by definition a postlexical context, so it must apply in the postlexical level. But because it is also affected by the internal morphology of words, for which information is postulated not to be available postlexically due to bracket erasure, it must also apply within the lexicon. I therefore provisionally adopt the most economic assumption, that it has an unrestricted domain and may apply at any level.

**Quantitative consequences of variable lexical phonology**

Suppose that we now modify the theoretical framework of lexical phonology to include the concept of the variable rule. What happens to the output of the system when such a rule is allowed to apply at more than one level of the derivation, provided its structural descriptions are met? For categorical
rules, the possibility of multilevel application merely entails that a rule applies at the earliest possible opportunity to a form that fulfills its structural description; the principle empirical consequence is that different feeding and bleeding relationships among rules may appear, depending on the level at which a word assumes a particular phonological form.

However, for rules that are variable in the sense of Labov (1969), some striking quantitative consequences emerge from this model. For the purposes of this discussion, I concentrate on the “context-free” properties of my model; that is, I ignore for the present the effects of variable conditioning factors. The model makes interesting empirical predictions about such factors, but they are left to a separate article dealing with “context-sensitive” aspects.

Suppose that a variable rule in lexical phonology has a fixed probability of applying whenever its structural description is met, regardless of derivational level. Let this probability be represented as \( p_a \). At a given level, it will apply to this proportion of forms, altering them so they no longer fulfill the relevant structural description. But it will not apply to the remainder, a fraction equal to \( 1 - p_a \), which I designate as the probability of retention, or \( p_r \). Note that all of these remaining forms still meet the structural description of the rule. Thus, at the next level of the derivation, if there is one, such forms will again be exposed to the rule, and once again a fraction equal to \( p_a \) of these will undergo the rule. Therefore, forms introduced early in the derivation will have multiple exposures to the rule, and the proportion of forms on which the rule has never operated will get smaller and smaller at each level. The proportion of forms to which the rule has not applied after \( n \) levels of the derivation will be \((1 - p_a)^n\) or, in other words, \( p_r^n \).

Therefore, for a variable rule that applies to different morphological classes of words (with different derivational histories), the model predicts that samples of these words should show an exponential progression of rates of nonapplication. For example, for a deletion rule that applied 50% of the time, words that met the structural description of the rule only at the end of the derivation and were subjected to the rule only once would have 50% of all forms retained, unaffected by the rule. Words that were subjected to the rule twice would have only 25% retentions, and those that were subjected to the rule three times would have a mere 12.5% retention rate.

Just such a situation obtains with \(-t,d\) deletion. I assume a form of the rule that looks like (1) (ignoring for the present variable conditioning, and adopting the bracketing assumptions of lexical phonology):

(1) \(-t,d\) Deletion

\[
\langle \text{variable, probability of application } = p_a \rangle
\]

\[
\{t,d\} \rightarrow \langle \emptyset \rangle / C_______
\]

Of our three categories, the regular past tense verbs will undergo deletion only once, after level 2 affixation and bracket erasure (i.e., postlexically). At
all earlier levels, they do not meet the structural description of the rule, because the final consonant cluster ending in -t or -d has not yet been built up by the morphology. Therefore, they should surface showing a deletion rate of only $p_a$ and a retention rate of $p_r$. At the other extreme, monomorphic words are subject to the rule three times: first, within the base morpheme; second, after level 1 affixation and bracket erasure; and third, after level 2 affixation. Therefore, such words should show a rate of retention of $p_r^3$.

By this reasoning, speakers who derive the past tense forms of semiweak verbs by level 1 affixation should expose these forms to the rule twice, after level 1 affixation and bracket erasure and again after level 2 affixation. Therefore, they should have a retention rate of $p_r^2$. But there is good evidence to suggest that not all speakers of English have this analysis of these forms. Several studies have found that the -t,d deletion rate in this class shows more interspeaker diversity than the other two. Guy and Boyd (1990) demonstrated an age-graded acquisitional pattern for this diversity. Younger speakers do not yet assign a distinct morphological analysis to these words and delete them at a high rate. For many of them, the semiweak verbs show deletion rates approximating those of monomorphic words, suggesting that they treat the final stop as an unanalyzed piece of the root morpheme. The morphological analysis postulated here (derivation at level 1) appears to be firmly adopted by speakers only when they are well into their adult years (and some may never adopt it). Based on these findings, the level 1 analysis of semiweak past tense forms is justifiable mainly for older adults; for younger speakers, the retention rate should be less than $p_r^2$ and indeed might approximate or fall below $p_r^3$.

**ANALYSIS AND DISCUSSION**

To test the exponential hypothesis on the data, one needs an estimate of the basic rate of retention ($p_r$). As a first approximation, one can use the observed raw frequencies of -t,d retention given in Table 1 as a basis for this estimation. The observed data actually provide three different estimates of the basic rate of retention, one derived from each sampled category. Starting with the observed rate of surface -t,d retention in monomorphic words, one takes the cube root to obtain an estimate of $p_r$. From the observed rate of retention in the semiweak words, one takes the square root, and for the regular past tense words, one simply sets $p_r$ equal to the observed percentage of retention.

**Pooled data**

These estimates of $p_r$ are given in Table 2 for the pooled data. The values obtained from monomorphic and regular past tense words are very close, which I take as confirming the hypothesis. The value derived from the data on semiweak verbs is somewhat lower than the other two, but this is not es-
TABLE 2. Estimates of the value of $p_r$

<table>
<thead>
<tr>
<th>Morphological class</th>
<th>Observed (surface) rate of retention</th>
<th>Estimated value of $p_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monomorphemic</td>
<td>61.9%</td>
<td>.852 (cube root of surface rate)</td>
</tr>
<tr>
<td>Semiweak past</td>
<td>66.1</td>
<td>.813 (square root of surface rate)</td>
</tr>
<tr>
<td>Regular past</td>
<td>84.0</td>
<td>.840</td>
</tr>
</tbody>
</table>

TABLE 3. -t,d retention in four adult speakers

<table>
<thead>
<tr>
<th>Morphological class</th>
<th>$N$ Total</th>
<th>$N$ Retained</th>
<th>% Retained</th>
<th>Estimated $p_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monomorphemic</td>
<td>357</td>
<td>273</td>
<td>76.5</td>
<td>.914</td>
</tr>
<tr>
<td>Semiweak past</td>
<td>26</td>
<td>20</td>
<td>76.9</td>
<td>.877</td>
</tr>
<tr>
<td>Regular past</td>
<td>98</td>
<td>90</td>
<td>91.8</td>
<td>.918</td>
</tr>
</tbody>
</table>

especially surprising. In the first place, this estimate is based on the smallest sample (only 56 tokens vs. 181 regular past tense words and 658 monomorphs), which makes it the most likely to be deviant. But in addition, as we have noted, previous work shows that deletion rates in this class are exceptionally high for younger speakers. Because the present sample includes three children and two young adults, it is expected that for the pooled data the rate of absence would be relatively high and the surface rate of retention relatively low. (Estimating $p_r$ by taking the square root in this category will understate its value if some unknown fraction of the tokens in the pool were actually exposed to the deletion rule three times.)

Because of the known high level of variability in these processes among younger speakers, it may be possible to obtain a cleaner test of the hypothesis by restricting our attention to the adult speakers in the sample, of whom there were four. These figures are given in Table 3.

For this population, the estimates of $p_r$ based on the monomorphemic and regular past tense data are almost identical. In fact, they are about as close as they can get given the sample size in the study. A best-fit estimate of $p_r$ may be obtained using a chi-square minimization technique; this yields a value of .91496. Using this figure to predict expected values for surface retention gives the results in Table 4.

Here, the difference between observed values and the values predicted by the present model is less than one for the monomorphemic and regular past classes and less than two for the semiweak class. Given that in the real world only integral numbers of retentions can be observed, the data for the regular past tense and monomorphemic classes are an essentially perfect fit to the
TABLE 4. Observed and expected values of -t,d retention and deletion in four adult speakers

<table>
<thead>
<tr>
<th>Morphological class</th>
<th>Observed</th>
<th>Expected</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Deletion</td>
<td>Retention</td>
<td>Deletion</td>
<td>Retention</td>
</tr>
<tr>
<td>Monomorphemic</td>
<td>84</td>
<td>273</td>
<td>83.6</td>
<td>273.4</td>
</tr>
<tr>
<td>Semiweak past</td>
<td>6</td>
<td>20</td>
<td>4.2</td>
<td>21.8</td>
</tr>
<tr>
<td>Regular past</td>
<td>8</td>
<td>90</td>
<td>8.3</td>
<td>89.7</td>
</tr>
</tbody>
</table>

(based on $p_r = .91496$)

Note: chi-square = .9343, 2 df, $p > .65$.

hypothesis that their ratio of retention is $p_r/p_r^3$. This portion of the model is therefore strongly confirmed. In the case of the semiweak verbs, however, the observed values still diverge somewhat on the low side. This may be mere statistical fluctuation, or it may be related to Guy and Boyd's (1990) findings that speakers do not tend to adopt the level 1 analysis of these words until after the age of 30. The two adults in our sample with the highest rates of retention for semiweak verbs were aged 30 and 55, whereas the other two were 22 and 23. The results for individual speakers are discussed in the next section, but subdividing the data this far gives numbers of semiweak verbs that are so small as to be rather unilluminating.

A test of the overall goodness-of-fit between model and data may be obtained using the chi-square statistic. Since the expected values in Table 4 are based on the exponential model (rather than on the usual null hypothesis of no relationship between the variables along the two axes of the table), the significance statistics ($p$ values) calculated from this procedure will measure the probability of obtaining such a distribution if the exponential hypothesis is true. Therefore, a good fit between model and data will be indicated by small chi-square figures and large $p$ values. The result obtained ($p > .65$) indicates a very good fit.

**Individual data**

The next step in this approach is to look at the values obtained for individual speakers. Thus far, I have used the pooled data from all speakers together, because it gives larger Ns and therefore better estimates of the values. But in view of the fundamental assumption of variable rule analysis mentioned at the outset (that speakers may have different basic rates of application of a rule), the speakers should be examined separately, to see if their ratios of retention in the three morphological classes follow the powers of their idiosyncratic value of $p_r$. With the smaller Ns, it is expected that the fit to the model will not be as good, especially in the case of the semiweak verbs, but this still gives a more detailed test of the exponential hypothesis.
A summary of the results for individual speakers in the sample is given in Table 5.

In this table, the column labeled "Estimated \( p_r \)" compares the cube root of the surface retention rate in monomorphemic words with the square root of the rate in semiweak verbs and the first root of the rate in regular past tense forms. The present model is best fit when, for each speaker, these three values are the same. The M and P values are consistently close for each speaker; the maximum difference between them is .084, and the average difference is .045. The next column gives a minimum chi-square estimate of the value of \( p_r \), that is, one that gives the best fit between observed and expected values for all the speaker’s data. The within-speaker variability of these three estimates of \( p_r \) (the best-fit estimate and the estimates based on the M and P class data) is small compared to the between-speaker variance.

The last two columns of Table 5 give the expected numbers of retentions,
calculated using the best-fit estimate for $p_r$, and the difference between this expected value and the observed value in the second column. Of the 21 comparisons (3 values for each of 7 speakers), 15 observations are within one token of the nearest integral number of expected retentions, and three more observations are within two tokens. The three remaining cases with more substantial divergences from the model are: the relatively low rate of retention in semiweak verbs by speaker #3, the relatively high rate of retention in this class by speaker #2, and the high (in fact, categorical) rate of retention in the regular past by speaker #8.

Another way to look at these data is to compare the predicted and observed rates of retention for each speaker; this is done in Figures 1 and 2. Figure 1 is a scattergram of the speakers' observed rates of retention in monomorphemic words plotted against the best-fit estimate of $p_r$. Values that fit the present model should fall close to the plotted curve, which gives the cube of $p_r$. All points fit this curve very well. In Figure 2, the speakers' observed rates of retention in regular past tense forms are plotted against their best-fit estimate for $p_r$. Here, values fitting the model should fall on the straight line ($p_r = p_r$). Again, the data points tend to conform to the model. Overall, these individual results are very good, considering the higher variability that arises when smaller $N$s are used.

One further point to note about the individual data is the age grading. Speakers tend to show a higher rate of retention as they get older. Figure 3 is a plot of the estimated value of $p_r$ by speaker's age. As with most language acquisition phenomena, the curve is steepest in childhood and adoles-
ence and tends to level off in adult life. This may actually represent the acquisition by adults of a more formal style, in which natural phonological rules are suppressed. The children in the sample may simply lack this stylistic option.
### Table 6. Chi-square and significance measures for the exponential model

<table>
<thead>
<tr>
<th>Speaker</th>
<th>Total chi-square</th>
<th>df</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>#6</td>
<td>.479</td>
<td>2</td>
<td>.79</td>
</tr>
<tr>
<td>#3</td>
<td>5.011</td>
<td>2</td>
<td>.08</td>
</tr>
<tr>
<td>#2</td>
<td>4.966</td>
<td>2</td>
<td>.08</td>
</tr>
<tr>
<td>#1</td>
<td>.130</td>
<td>2</td>
<td>.94</td>
</tr>
<tr>
<td>#8</td>
<td>8.461</td>
<td>2</td>
<td>.01</td>
</tr>
<tr>
<td>#7</td>
<td>.477</td>
<td>2</td>
<td>.79</td>
</tr>
<tr>
<td>#4</td>
<td>.487</td>
<td>2</td>
<td>.78</td>
</tr>
<tr>
<td>Total chi-square (all speakers):</td>
<td>20.011</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Statistical Testing of the Model

As the model can predict expected values for each category, it is possible to calculate a chi-square test of significance for these data, as was done in Table 4 for the pooled data. The expected retention values derived in Table 5, together with the corresponding expected values for deletion, yield a 3 × 2 contingency table with two degrees of freedom for each speaker. As before, a good fit between model and data is indicated by large p values. The calculated values are summarized in Table 6 (note that the “Total chi-square” values are the sum, for each speaker, of the contributions of all six contingency cells: both retentions and deletions for each of the three morphological classes). Using .05 as the criterion, the exponential hypothesis is rejected only for speaker #8 (p = .01). This one rejection is partly because of a low N for semiweak verbs, but this speaker’s principal divergent characteristic has already been noted: categorical retention in past tense forms. Two of the other speakers (#3 and #2) have relatively low p values (p = .08) due to their anomalous retention rates in the semiweak verbs. But these anomalies are not sufficient to reject the exponential hypothesis. Four speakers have high p values (.78 or above), indicating close conformity with the model.

Thus, these significance tests do not reject the exponential hypothesis. But do the results allow a stronger claim of confirmation? The problem here is that there are many possible mathematical relationships that might exist among these values (indeed, an infinite number), and one cannot try to confirm the proposed relationship by rejecting all the others. The only sensible approach is to test the proposed relationship against particular alternatives with plausible theoretical interpretations. In this case, the obvious alternative is the traditional variable rule treatment of these facts sketched earlier: the three morphological categories form a factor group constraining the rule, and each category is associated in the logistic model with a factor value that is essentially independent of the other two.
TABLE 7. Expected retentions, chi-square and significance measures for the logistic model

<table>
<thead>
<tr>
<th>Speaker</th>
<th>M</th>
<th>S</th>
<th>P</th>
<th>Total chi-square</th>
<th>df</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>#6</td>
<td>25.54</td>
<td>3.14</td>
<td>9.34</td>
<td>.929</td>
<td>2</td>
<td>.66</td>
</tr>
<tr>
<td>#3</td>
<td>25.60</td>
<td>3.73</td>
<td>20.70</td>
<td>4.131</td>
<td>2</td>
<td>.16</td>
</tr>
<tr>
<td>#2</td>
<td>84.11</td>
<td>9.25</td>
<td>31.63</td>
<td>5.814</td>
<td>2</td>
<td>.06</td>
</tr>
<tr>
<td>#1</td>
<td>76.29</td>
<td>11.96</td>
<td>22.73</td>
<td>0.286</td>
<td>2</td>
<td>.88</td>
</tr>
<tr>
<td>#8</td>
<td>80.74</td>
<td>2.51</td>
<td>29.73</td>
<td>8.060</td>
<td>2</td>
<td>.02</td>
</tr>
<tr>
<td>#7</td>
<td>36.48</td>
<td>5.52</td>
<td>19.00</td>
<td>0.774</td>
<td>2</td>
<td>.71</td>
</tr>
<tr>
<td>#4</td>
<td>78.31</td>
<td>0.86</td>
<td>18.81</td>
<td>0.780</td>
<td>2</td>
<td>.71</td>
</tr>
<tr>
<td>Total chi-square (all speakers):</td>
<td></td>
<td></td>
<td></td>
<td>20.774</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Varbrul factor values (for deletion):
Input = .26
M = .64  S = .55  P = .32
#6 = .79  #3 = .74  #2 = .56  #1 = .38  #8 = .32  #7 = .40  #4 = .28

In constructing this test, I adhere to the first two assumptions of variable rule analysis previously cited: speakers have the same conditioning factor values but may differ on a single parameter characterizing their individual propensity to use the rule. I therefore conduct a variable rule analysis using two factor groups: one containing a factor for each of the seven speakers, and one for morphological class (M, S, or P), giving a total of 21 cells. This derives a single set of constraint values for morphological class from the pooled data but allows each speaker one parameter to represent their personal behavior. Then we can use the standard logistic model to generate predicted values of deletions and retentions for each speaker in each category, and a chi-square test of fit may be calculated for comparison with the results for the exponential model given in Table 6.

The results of this test of the logistic model appear in Table 7. Expected retentions for each of the three categories for each speaker may be compared with the values in Table 5 for observed retentions and expected retentions from the exponential model. It will be noted that the logistic predictions give a worse fit to the observed data in 15 of the 21 cells. The individual chi-square totals indicate that the logistic model gives the worse fit for five of the seven speakers, and the $p$ values show that it is rejected for the same speaker (#8) that the exponential model failed to fit.

The total chi-square in this table (across all cells for all speakers) differs only slightly from the comparable figure for the exponential model, indicating that both models fit the data about equally well overall. But, strikingly, the logistic model requires more parameters without achieving any improvement in fit! While the exponential model predicts all cells using only the 7
speaker parameters, the logistic model uses 9 independently estimated values (7 - 1 = 6 from the speaker group, 3 - 1 = 2 from the morphological factor group, plus one input parameter, for a total of 9). Normally, adding more parameters will improve goodness-of-fit, so in this case one must conclude that the architecture of the logistic model is simply inefficient at capturing the relationship that actually obtains in the data. In this case, the inefficiency lies in the assumption of an essentially linear relationship among the morphological values for all speakers. That relationship is more accurately characterized as an exponential one.

**FORMULATING THE RULE**

The exponential model is therefore successful at accounting for the quantitative facts in some detail. But two qualitative problems remain to be resolved. First, in the initial formulation of the -t,d deletion rule as in (1), I ignored the following-segment effect, which has been demonstrated in all studies of the phenomenon: less sonorant initial segments of following words (e.g., obstruents) promote deletion and more sonorant ones (e.g., vowels and glides) inhibit it. Including this as a constraint would give a rule something like (2) (where sonority is treated as a continuous variable). But in the architecture adopted here from lexical phonology, such constraints can only apply to postlexical rules, because in the lexicon there are, by definition, no following words, and no cross-word-boundary effects. How then can we capture this constraint without abandoning the most attractive feature of the present analysis: the utilization at multiple derivational levels of a single rule with a fixed rate of application?

(2) \[-t,d \text{ Deletion, incorporating following-segment effect:}\]

\[-t,d \rightarrow \langle \varnothing \rangle / C \quad \#\# \langle -x \text{ sonority} \rangle\]

The second remaining problem is that the model appears to predict certain nonoccurring forms. If deletion can apply at level 1 to forms that may subsequently undergo affixation at level 2, then verbs like test and land would be predicted to show derived forms like *tessing, *tessed, *lanning, *lanned, when in fact they do not.8 Furthermore, the invariant occurrence of tested and similar forms with the syllabic form /id/ of the past tense affix suggests that the -t,d is always available to the morphophonemic operations, because this form of the affix is triggered only by a root-final apical stop. How can invariant retention at level 2 in these cases be reconciled with my hypothesis of variable deletion at level 1?

A solution to both of these problems depends on decomposing -t,d deletion into several separate processes. The first problem would be resolved if the following-segment effect is not a constraint directly on -t,d deletion but on some other process that interacts with it postlexically (by feeding or bleed-
TABLE 8. -t,d absence by following context

<table>
<thead>
<tr>
<th>Following segment</th>
<th>Varbrul factor value</th>
<th>Syllable onset conditions in English</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obstruent</td>
<td>.66</td>
<td>*ts-, *tk-, *tn-, etc.</td>
</tr>
<tr>
<td>/l/</td>
<td>.80</td>
<td>*l-</td>
</tr>
<tr>
<td>Glide</td>
<td>.57</td>
<td>tw + front vowel, tyu- (some dialects)</td>
</tr>
<tr>
<td>/r/</td>
<td>.42</td>
<td>r-</td>
</tr>
<tr>
<td>Vowel</td>
<td>.19</td>
<td>ta-, etc.</td>
</tr>
<tr>
<td>Pause</td>
<td>.37</td>
<td></td>
</tr>
</tbody>
</table>

ing it); this would allow the -t,d rule to have the same form lexically and postlexically. What process is available that can apply postlexically, is sensitive to sonority, can interact with -t,d deletion, and is independently motivated? An obvious answer is syllabification. Suppose that word-final stops may, when they satisfy possible-onset constraints (i.e., before vowels and certain glides and liquids), be syllabified as the initial segment in the following syllable (e.g., band-aid is syllabified as /ban.daid/) and consequently become ineligible for deletion. This bleeding of otherwise deletable stops when they are followed by segments of high sonority would appear to lower the rate of -t,d deletion in these conservative environments, thus giving the correct surface output without a direct following-segment constraint on deletion.

This hypothesis entails a novel prediction. If syllabification is responsible for the following-segment effect on -t,d deletion, then the prior practice (in works such as Guy, 1980; Labov, 1989, etc.) of putting following /l/ and /r/ together as sharing a common sonority level was mistaken. The prohibition against *tl- and *dl- onsets is one of the best-known phonotactic constraints in English, whereas tr- and dr- onsets abound in the language. Therefore, final -t,d should be blocked from rightwards-syllabification before /l/ but not before /r/. This prediction is confirmed in my data, as shown in Table 8. Deletion before /l/ is as likely as before obstruent consonants, but before /r/ it is rare.

The following-segment constraint may therefore be justifiably treated as a constraint on syllabification; its properties follow directly from this formulation. In this analysis, syllabification may itself be a variable rule, but when it operates on a final stop, that segment cannot subsequently be affected by the -t,d deletion rule. With this treatment, the following-segment constraint can be removed from the statement of the -t,d deletion rule, resolving the first problem.

Extending this analysis into the lexicon also helps solve the second problem. Cases like testing, landing occur because the addition of a following vowel at level 2 regularly licenses the formation of a syllable to which the stop can adhere as an onset. But still we require some mechanism by which "de-
leted" segments are later restored under these favorable conditions. This suggests a representation in which the final stops are structurally detached in some way but not completely removed. The obvious candidate is an autosegmental treatment, where -t, d "deletion" merely delinks the segment from higher phonological structure without immediately removing it from the melodic stream. The segment is therefore still available to trigger correct affixation and to be relinked by syllabification to a following syllable attached later in the derivation. But when such a relinking does not occur, the segment is ultimately deleted, possibly by a process of "stray erasure" that removes any segments that do not have a proper structural position upon exiting the phonology.

In this analysis, the general consonant cluster simplification rule can be formulated as (3):

\[
(3) \text{Consonant cluster simplification} \\
\langle \text{variable, unmarked domain of application} \rangle
\]

This rule applies after syllabification, and prior to bracket erasure, at all phonological levels. Any final consonant left unlinked by this operation, and not subsequently relinked by another process, will not receive phonetic realization.

CONCLUSIONS

The predictions of this variationist version of lexical phonology that I am proposing are very well-supported by the data. The exponential model of morphological constraints is more parametrically parsimonious, and actually fits the data better than a standard logistic treatment. Previous practice in variable rule analysis has generally involved the following assumptions about morphological constraints. They represent boundary conditions or functional restrictions on the phonological rules; they can best be quantitatively modeled as factors in a logistic analysis; they have the property of such factors that their values may vary independently of the others in the factor group and independently of the input value. The present model contradicts all of these points. Morphological classes are differentiated by the rules because of their different derivational histories, and they can be quantitatively modeled as the powers of a base rate of application of a rule that may apply more than once; therefore, the rate of application in each class is not independent of the other classes.
Crucially, the model predicts that morphological constraints on variation should differ in these quantitative aspects from purely phonological constraints. A constraint like the preceding-segment effect on \( -t,d \) deletion (more deletion after obstruents than sonorants) is not due to differences in derivational history but to phonetic and phonological properties of segments and phonotactic principles of English. Hence, there is no expectation that such constraints should be exponentially ordered; rather, they are the kinds of constraints that the logistic framework is expressly designed to model.

This points the way to future investigations of the theory. The phonological constraints on this rule should be examined to see if they differ in this way from the morphological ones. Other variable rules subject to morphological conditioning (e.g., /s/ deletion in Caribbean Spanish and Brazilian Portuguese) should be investigated in this light. And, of course, postlexical constraints must be examined to see if they differ from word-internal ones. However, these are all topics for other articles.

One further feature of the present model deserves explicit statement. I am presupposing a relationship between abstract theories of competence and observable properties of production that is not always postulated in linguistics — namely, that the theory is in some ways isomorphic with the processes of production. If lexical phonology is conceived merely as a theory of competence that may be quite different from what goes on in the minds of speakers when they speak, then there is no basis for making the predictions I have made. But, of course, without an explicit account of the relationship between theory and data, no theory is ever empirically testable. I take the view that linguistics should strive to develop psychologically valid and empirically testable theories of language use; my assumption of isomorphism is merely the first and simplest step in developing such an interpretation of a theory.

In the case at hand, it is possible that there are other theories, as yet unthought of, that would explain these facts in another way and predict some other mathematical relationship among the several morphological conditions that better fits the data. But for the present, the data are best accounted for in terms of different integral numbers of applications of a unitary variable rule that applies repeatedly in morphological derivations, yielding an exponential relation among deletion rates in the various morphological classes.

NOTES

1. Lexicostatistics and glottochronology are notable exceptions to this generalization.
2. Explanatory value also depends on other criteria, such as psychological and social validity, and utility in achieving Chomsky's goal of "evaluating alternative proposed grammars" (1965:31); my focus is on quantitative explanation.
3. Adult (L2) learners of English appear to show a very different pattern (see Bayley, 1991).
4. The strong verbs of English, of course, do not systematically terminate in the past tense in consonant clusters with a final apical stop; in those that do, like found, held, the stop is part of the root morpheme and undergoes deletion at a rate not significantly different from comparable underived forms.
5. The replacive class of send-sent, etc., was coded as M for this study, under the assumption that such verbs fulfill the structural description of \( -t,d \) deletion from the earliest lexical insertion.
6. Note that the factor probabilities obtained from Varbrul analysis will not show the exponential relationship, because they have undergone the logistic transform and have been adjusted to form a balanced distribution centered on the .5 value.

7. It is actually rather surprising to obtain a fit this good, given normal expectations of sampling error and statistical fluctuation. But comparable results are now being obtained from other data sets. Since my presentation of these findings at NWAV-XIX, Otto Santa Ana has found a similar close fit to the predictions of the exponential model in his corpus of data from 45 Chicano English speakers. His pooled results (from Santa Ana, 1991) are as follows:

<table>
<thead>
<tr>
<th>Class</th>
<th>N</th>
<th>% Retained</th>
<th>Estimated (p_r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>3467</td>
<td>.41</td>
<td>.744</td>
</tr>
<tr>
<td>S</td>
<td>280</td>
<td>.61</td>
<td>.781</td>
</tr>
<tr>
<td>P</td>
<td>830</td>
<td>.75</td>
<td>.750</td>
</tr>
</tbody>
</table>

8. Such forms are occasionally attested for some speakers, mainly children, but they do not systematically occur in adult dialects I am familiar with.

9. This syllabification analysis of the following-segment effect does not, however, explain the conservative effect of following pause in some dialects (see Guy, 1980). If my analysis is correct, the following-pause effect must be a constraint on some other postlexical process, such as the assignment of aspiration or other release types to stops in prepausal environment.

10. A precise statement of syllabification awaits further work. Postlexically, there is variable deletion of \(-t,d\) before vowels, but word-internally there is nearly categorical retention in this context, even though other contexts are variable (e.g., restless, respectful, restroom).

11. Specifically, the exponential model predicts that the effect of word-internal constraints (such as the preceding segment effect on \(-t,d\) deletion) will be magnified in words of the M and S classes through repeated applications in the lexicon, while the effect of external constraints (e.g., following segment) will be constant for all morphological classes, since they operate only once, at the postlexical level. Work in progress by the author appears to confirm these predictions.

REFERENCES


**APPENDIX**

*Social characteristics of consultants*

<table>
<thead>
<tr>
<th>Consultant no.</th>
<th>Age</th>
<th>Ethnicity</th>
<th>Sex</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22</td>
<td>White</td>
<td>F</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>Black</td>
<td>F</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>Samoan</td>
<td>F</td>
</tr>
<tr>
<td>4</td>
<td>55</td>
<td>White</td>
<td>M</td>
</tr>
<tr>
<td>6</td>
<td>9</td>
<td>Chicano</td>
<td>M</td>
</tr>
<tr>
<td>7</td>
<td>30</td>
<td>White</td>
<td>F</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
<td>White</td>
<td>M</td>
</tr>
</tbody>
</table>